國立臺北科技大學

106 學年一學期電機系博士班資格考試

最佳控制 試題

注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 可使用非程式型計算機。
3. 請按順序標明題號作答，不必抄題。
4. 全部答案均須答在試卷答案欄內，否則不予計分。
5. 考試時間：二小時。

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1. (25%) Consider the consistent system $Ax = b$ which many solutions, where $A \in \mathrm{R}^{n \times m}$, $n < m$, and AA^T is nonsingular. Find the minimum norm solution $\tilde{x}$ such that $\|x\|^2$ is minimized. (Note that you have to check the Hessian.)

2. (25%) Consider the parallel RL circuit indicated in Figure 1 with $R = 1\Omega$ and $L = 1\mathrm{H}$. Let the state variable be $x(t) = i_L(t)$ and the control current input be $u(t) = i_s(t)$. Find $u^*(t)$ to minimize the cost function

$$J = x^2(5) + \frac{1}{2}\int_0^5 \left[3x^2(t) + u^2(t) \right] dt \,.$$
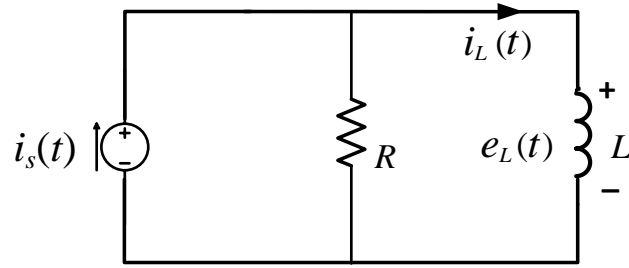
Figure 1

3. Consider the system $\dot{x}_1 = x_2$, $\dot{x}_2 = u$ with the performance index

$$J = \frac{1}{2}\int_0^\infty \left(x_1^2 + 2x_2^2 + 2x_1 x_2 + u^2 + 2(x_1 + x_2)u \right)dt .$$

(a) (15%) Find the optimal control law $u^*(t)$.

(b) (10%) Compute the optimal cost J^* for $x_1(0) = -1$, $x_2(0) = -2$.

4. (25%) Consider the discrete-time control system

$$x(k+1) = 0.25x(k) + u(k), \quad x(0) = 4$$

with the performance index: $J = \frac{1}{2}\sum_{k=0}^{\infty}[2x^2(k) + u^2(k)]$. Determine the

optimal control $u^*(k)$ and the optimal cost J^*.