

# 國立臺北科技大學

九十八學年第一學期電機系博士班資格考試

## 最佳控制試題

第一頁 共一頁

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### 注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間：二小時。

1. Consider the system  $Ax = b$  which has many solutions, where  $A \in \mathbb{R}^{n \times m}$ ,  $n < m$ , and  $AA^T$  is nonsingular. Find the minimum norm solution  $\tilde{x}$ .

2. Given the ARE

$$PA + A^T P + C^T C - PBR^{-1}B^T P = 0.$$

The associated Hamiltonian matrix is  $M = \begin{bmatrix} A & -BR^{-1}B^T \\ -C^T C & -A^T \end{bmatrix}$ . Show that  $M$  has no pure

imaginary eigenvalues under the controllability of  $(A, B)$  and observability of  $(C, A)$ .

3. Let  $\dot{x}_1 = x_2$ ,  $\dot{x}_2 = u$ ,  $J = \frac{1}{2} \int_0^\infty (x_1^2 + x_2^2 + u^2 + 2x_1 u) dt$ . Find the optimal control law  $u^*$ .

4. Consider the first-order plant  $\dot{x} = x + u$ ,  $y = x$ . The control law  $u = -5x$  is an optimal law for this plant.

(a) Calculate the observer gain adopted as the *steady-state* Kalman filter gain with the weighting matrices  $Q=1$  and  $R=1$ .

(b) Find the transfer function of the observer-based controller (from  $y$  to  $u$ ).