國立臺北科技大學

九十六學年第二學期電機系博士班資格考試

隨機程序

填學生證號碼

注意事項：

1. 本試題共【5】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間:二小時。

1. (a) (6pt) Prove the markov's inequality:

   (Marcov inequality) Let U be a nonnegative random variable, then

   $$P_r\{U \ge \alpha\} \le \frac{EU}{\alpha} \quad \text{for} \quad \alpha > 0.$$

   (b) (6pt) Let X, Y are two random variable with joint pdf

   $f_{X,Y}(x,y) = \frac{1}{\pi}\exp\left\{-\frac{1}{2}(x^2+y^2)I_A(x,y)\right\}$ where $A = \{(x,y): xy > 0\}$. Find the marginal pdf f_X.

   (c) (4pt) Is X in (b) a Gaussian random variable? Explain your answer.

   (d) (4pt) Is $\bar{X} = [X\ Y]^T$ in (b) a Gaussian vector? Explain your answer.

2. (a) (4pt) (2pt) Give the definition of a random variable on a probability space $(\Omega, \boldsymbol{F}, P)$.

   (b) (6pt) Let $\Omega = (0,1], \boldsymbol{F} = B(0,1]$, and P be arbitrary probability measure on $(\Omega, \boldsymbol{F})$. Consider two random variables $X(\omega) = I_{(0,1/4]}(\omega)$ and

   $Y(\omega) = I_{(0,1/4]}(\omega) + 2I_{(3/4,1]}(\omega)$ where $I_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases}$ is a indicator function.

   Find $\sigma(X)$, $\sigma(Y)$ and $\sigma(X) \cap \sigma(Y)$ where $\sigma(X)$ is the $\sigma-$field generated by random variable X.

   (c) (5pt) Give the definition of independence of two random variables X and Y on the same probability space $(\Omega, \boldsymbol{F}, P)$.

   (d) (5pt) Is it possible that the random variables X and Y in part (b) are independent under some porbability? If the answer is yes, give an example. If the answer is no, prove it.

3. (a) (10pt) Draw the relationship diagram for a random sequence $\{X_n; n \in \mathbb{N}\}$ converges in the following modes: point wise convergence, almost sure convergence, convergence in probability, convergence in r_th mean, convergence in distribution.

   (b) (5pt) Show that if $X_n \xrightarrow{P} X$, than $X_n \xrightarrow{d} X$.

   (c) (5pt) Let $X_1, X_2, \ldots$ be i.i.d. random variables. Define $Y_n = \max_{i \le n} X_i$. Determine whether the sequence $\{Y_n; n = 1, 2, \ldots\}$ of random variables converges in distribution.

4. Let $\{X_n; n = 1, 2, \ldots\}$ be an i.i.d. binary alphabet process with $P(X_n = 1) = 1/3$, $P(X_n = 0) = 2/3$ and let N_t be the Poisson counting process with rate $\lambda > 0$. A continuous time random process is defined by $Y(t) = \sum_{k=1}^{2N_t} X_k$.

   (a) (6pt) Find the mean and variance of X_n.

   (b) (6pt) Find the expectation of $Y(t)$.

   (c) (4pt) Find the covariance function of $Y(t)$.

   (d) (4pt) Find the characteristic function of $Y(t)$.

5. (a) (4pt) Give the definition of a wise-sense stationary process.

   (b) (4pt) Give the definition of a stationary process.

   (c) (6pt) Give an example of a random process such that it is stationary and dependent. Explain your answer.

   (d) (6pt) Let $\{X_t; t \in I\}$ be a wise-sense stationary process and $f: \mathbb{R} \to \mathbb{R}$ is a measurable function. Is the random process $\{f(X_t); t \in I\}$ wise-sense stationary? Explain your answer.