國 立 臺 北 科 技 大 學

九十六學年第一學期電機系博士班資格考試

隨機程序(公告用)

填 學 生 證 號 碼　　　　　　　　　　　　　　　　第二頁　共二頁

注意事項：

1. 本試題共【5】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間:二小時。

1. (a) (6pt) Consider the probability space $(\Omega, \mathcal{F}, P)$. Here $\Omega = (0,1]$, P is arbitrary, and $\mathcal{F} = \{\phi, \Omega, A, A^C\}$, where $A = (0,1/3]$. Is the function X defined by $X(\omega) = I_A(\omega)$ a random variable on $(\Omega, \mathcal{F}, P)$? Justify your answer.
   (b) (6pt) $\{X_k, k \in \mathbb{N}\}$ is an i.i.d. sequence of random variables such that $E\left[|X_k|\right] < 1$. Show that the product $\prod_{k=1}^{n} X_k$ converges to zero in probability as n tends to infinity.
   (c) (6pt) The characteristic function of a random vector $\bar{X} = [X_1, X_2]$ is given by $\Phi_{\bar{X}}(u_1, u_2) = \exp\{-3u_1^2 + u_1 u_2 - u_2^2\}$. Determine the MMSE linear estimator of X_1 given X_2.

2. (a) (12pt) Let X be a random variable on a probability space $(\Omega, \boldsymbol{F}, P)$. State and prove the basic three properties of the cdf $F_X(x) \triangleq P(X \le x)$.
   (b) (8pt) Let $\Omega = (0,1]^2, \boldsymbol{F} = B\left((0,1]^2\right)$ and P be a probability measure with uniform density $(\omega_1, \omega_2) \in (0,1]^2$ $f(\omega_1, \omega_2) = 1$. Define the random variables $U(\omega_1, \omega_2) = \omega_1 + \omega_2$, $V(\omega_1, \omega_2) = \omega_1 - \omega_2$. Compute $F_{UV}(1,0)$.

3. (a) (9pt) Give the definitions (the requirements) of the items $\Omega, \boldsymbol{F}, P$ in a probability space $(\Omega, \boldsymbol{F}, P)$.
   (b) (6pt) Let $\Omega = (0,1]$, and let G consists of subsets of Ω that are either finite or cofinite (A is cofinite if A^C is finite). Is the collection G a field on Ω? Explain your answer? ("A set is finite" means this set contains only finite points)
   (c) (7pt) Is G a $\sigma -$ field on Ω? If yse, prove it. If no, find $\sigma(G)$.

4. (a) (10pt) Draw the relationship diagram for a random sequence $\{X_n; n \in □\}$ converges in the following modes: point wise convergence, almost sure convergence, convergence in probability, convergence in r_th mean, convergence in distribution.
   (b) (4pt) Show that if $X_n \xrightarrow{Lr} X$ for $r \geq 1$, than $X_n \xrightarrow{P} X$.
   (c) (6pt) Give an example $\{X_n; n \in □\}$ such that $X_n \xrightarrow{P} X$, but $X_n \not\xrightarrow{Lr} X$ and $X_n \not\xrightarrow{a.s.} X$. Justify your answer.

5. Let $\{X_n; n = 1,2,\ldots\}$ be an i.i.d. Gaussian process with marginal pdf $N(0,1)$ and let N_t be the Poisson counting process. A continuous time random walk can be defined by $Y(t) = \sum_{k=1}^{N_t} X_k$.
   (a) (6pt) Find the expectation, covariance function of $Y(t)$.
   (b) (6pt) Find the characteristic function of $Y(t)$.
   (c) (4pt) Is $Y(t)$ a Gaussian process? Explain your answer.
   (c) (4pt) Is $Y(t)$ a wide-sense stationary process? Explain your answer.