

國立臺北科技大學

102 學年第一學期電機系博士班資格考試

最佳控制 試題

第一頁 共一頁

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注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 可使用非程式型計算機。
3. 請按順序標明題號作答，不必抄題。
4. 全部答案均須答在試卷答案欄內，否則不予計分。
5. 考試時間：二小時。

1. (25%) Consider the quadratic performance index $L(x,u) = \frac{1}{2} \left(\frac{x^2}{a^2} + \frac{u^2}{b^2} \right)$

with the equality constraint $f(x,u) = x + mu - c = 0$. Find the optimal performance index value L^* .

2. (25%) Consider the linear system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$, $x(0)$ given,

and the cost function $J = \frac{1}{2} x^T(T)S(T)x(T) + \frac{1}{2} \int_{t_0}^T [x^T Q(t)x + u^T R(t)u] dt$

where weighting matrices S and Q are symmetric and semi-positive definite,

and R is symmetric and positive definite. Show $\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$

which is called the Hamiltonian system of considered optimization problem with Lagrange multiplier λ .

3. (25%) Consider the linear time-invariant system and the cost function

$$\dot{x}(t) = Fx(t) + Gu(t), \quad x(0) \text{ given,}$$

$$J = \frac{1}{2} \int_0^{\infty} e^{6\alpha t} \left\{ \left[x^T(t) Q x(t) \right]^2 + \left[u^T(t) R u(t) \right]^2 \right\} dt$$

where Q and R are constant positive matrices. Find a related linear system and cost function where the integrand in the cost function does not specifically depend on the time t .

4. (25%) Let the system be $\dot{x}_1 = x_2$, $\dot{x}_2 = u$ with

$$J = \frac{1}{2} \int_0^{\infty} (2x_1^2 + x_2^2 + 2x_1x_2 + u^2 + 2(x_1 + x_2)u) dt.$$

Find the optimal control law u^* .