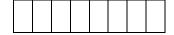
國立臺北科技大學

102 學年第一學期電機系博士班資格考試

最佳控制 試題



with Lagrange multiplier λ .

- 注意事項:
 1. 本試題共【4】題,配分共100分。
 2. 可使用非程式型計算機。
 3. 請按順序標明題號作答,不必抄題。
 4. 全部答案均須答在試卷答案欄內,否則不予計分。
- 1. (25%) Consider the quadratic performance index $L(x,u) = \frac{1}{2} \left(\frac{x^2}{a^2} + \frac{u^2}{b^2} \right)$ with the equality constraint f(x,u) = x + mu - c = 0. Find the optimal performance index value L^* .
- 2. (25%) Consider the linear system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$, x(0) given, and the cost function $J = \frac{1}{2}x^T(T)S(T)x(T) + \frac{1}{2}\int_{t_0}^T \left[x^TQ(t)x + u^TR(t)u\right]dt$ where weighting matrices S and Q are symmetric and semi-positive definite, and R is symmetric and positive definite. Show $\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$ which is called the Hamiltonian system of considered optimization problem

3. (25%) Consider the linear time-invariant system and the cost function

$$\dot{x}(t) = Fx(t) + Gu(t)$$
, $x(0)$ given,

$$J = \frac{1}{2} \int_0^\infty e^{6\alpha t} \left\{ \left[x^T(t) Q x(t) \right]^2 + \left[u^T(t) R u(t) \right]^2 \right\} dt$$

where Q and R are constant positive matrices. Find a related linear system and cost function where the integrand in the cost function does not specifically depend on the time t.

4. (25%) Let the system be $\dot{x}_1 = x_2$, $\dot{x}_2 = u$ with

$$J = \frac{1}{2} \int_0^\infty \left(2x_1^2 + x_2^2 + 2x_1x_2 + u^2 + 2(x_1 + x_2)u \right) dt.$$

Find the optimal control law u^* .