國 立 臺 北 科 技 大 學

106 學年第二學期電機系博士班資格考試

最 佳 控 制 試題

注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 可使用非程式型計算機。
3. 請按順序標明題號作答，不必抄題。
4. 全部答案均須答在試卷答案欄內，否則不予計分。
5. 考試時間：二小時。

1. Consider the cost function $L = ax^2 + bxu + cu^2$ with the constraint $x + mu + d = 0$ where a, b, c, m, and d are positive scalar constants.

   (a) (15%) Find the scalar u^* that yields an extremal value of L

   (b) (10%) Is $L(u^*)$ a minimal value or a maximal value? Prove it.

2. (25%) Given the ARE $AX + A^T X + Q + XRX = 0$. The associated Hamiltonian matrix is $M = \begin{bmatrix} A & R \\ -Q & -A^T \end{bmatrix}$. Show that if λ is one of the eigenvalues of M, so is $-\lambda$.

3. (25%) Consider the RLC circuit indicated in Figure 1 with $R = 1\,\Omega$, $C = 1$ F, and $L = 1$ H. Define state variables as $x_1(t) = e_c(t)$ and $x_2(t) = i_L(t)$. $u(t) = i_s(t)$ is the control input (dependent current source). Find $u^*(t)$ to

minimize the cost function $J = \dfrac{1}{2}\displaystyle\int_0^\infty \left[x_1^2(t) + u^2(t) \right] dt$.
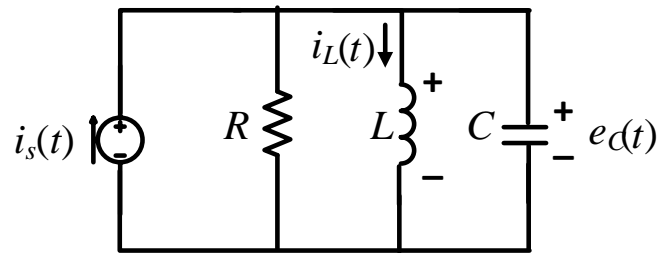

Figure 1

4. (25%) Consider a discrete-time control system defined by

$$x(k+1) = -2x(k) + u(k), \quad x(0) = 1.$$

Determine the optimal control $u(k) = -Kx(k)$ and the resulted optimal cost

for the performance index: $J = \dfrac{1}{2}\displaystyle\sum_{k=0}^\infty [x^2(k) + u^2(k)]$.