國 立 臺 北 科 技 大 學

九十七學年第一學期電機系博士班資格考試

現代控制理論試題

注意事項：

1. 本試題共【5】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間:二小時。

1. (30%) Apply an equivalence transformation $x(t) = P(t)z(t)$, where $P(t) \in \mathbf{R}^{n \times n}$, to the linear time-varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

(a) Determine the system matrices $\overline{A}(t), \overline{B}(t), \overline{C}(t),$ and $\overline{D}(t)$ of the system with state $z(t)$.

$$\dot{z}(t) = \overline{A}(t)z(t) + \overline{B}(t)u(t)$$
$$y(t) = \overline{C}(t)z(t) + \overline{D}(t)u(t)$$

(b) Justify that the state transition matrix of $\overline{A}(t)$ is $\Phi_{\overline{A}}(t,\tau) = P^{-1}(t)\Phi_A(t,\tau)P(\tau)$, where $\Phi_A(t,\tau)$ is the state transition matrix of $A(t)$.

(c) Show that the linear state equation $\dot{x}(t) = A(t)x(t)$, $x(t_0) = x_0$ is *uniformly stable* iff the state equation $\dot{z}(t) = \overline{A}(t)z(t)$ is *uniformly stable* given that $P(t)$ is a Lyapunov transformation.

2. (25%) Consider the linear time-invariant system

$$\begin{cases} \dot{x} = Ax + bu \\ y = cx \end{cases} \quad \text{where } A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \text{ and } c = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}.$$

(a) Justify that the system is neither controllable nor observable.

(b) Determine a state variable change with transformation matrix P to yield a special form that displays the controllable subsystem and uncontrollable subsystem. Find the corresponding system matrices and identify the controllable and uncontrollable states.

(c) Determine a state variable change with transformation matrix Q to yield a special form that displays the observable subsystem and unobservable subsystem. Find the corresponding system matrices and identify the observable and unobservable states.

3. (10%) Show that The linear system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

is *controllable on* $[t_0, t_f]$ if the $n \times n$ *controllability Gramian*

$$W(t_0, t_f) \equiv \int_{t_0}^{t_f} \Phi(t_0, t)B(t)B'(t)\Phi'(t_0, t)dt$$

is invertible.

4. (15%) Consider the linear time-varying system

$$\dot{x}(t) = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} x(t) + \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} u(t) \quad ; \quad t \in [t_0, t_f]$$

where $a_1 \neq a_2$, $b_1(t) = e^{a_1 t}$, and $b_2(t) = e^{a_2 t}$.

Determine if the system is controllable on $[t_0, t_f]$ or not.

5. (20%) Consider the linear time-invariant system

$$\begin{cases} \dot{x} = Ax + bu \\ y = cx \end{cases} \quad \text{where } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ and } c = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}.$$

Build an observer-based controller to stabilize the process such that the eigenvalues of the observer and the closed-loop state equation are given by $\{-1, -2, -3\}$ and $\{-1, -1, -1\}$ respectively.