國 立 臺 北 科 技 大 學

九十八學年第二學期電機系博士班資格考試

現代控制理論 試題

注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間:二小時。

1. (20%) Given a causal system with $y(t) = g(t) * u(t)$, where the star denotes convolution. Show that the Laplace transform of $y(t)$ can be expressed as $\hat{y}(s) = \hat{g}(s) \cdot \hat{u}(s)$, where $\hat{g}(s)$ is the transfer function of the system.

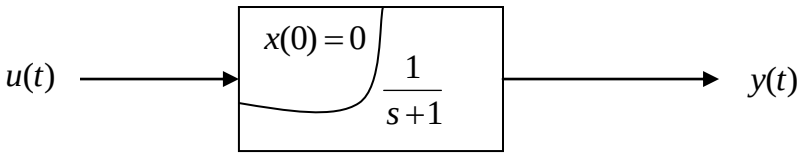
2. (30%) Consider the following linear time-invariant system.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Give the discretized system for the case $u[t] = u[kT]; t \in [kT, (k+1)T), k = 0,1,2,\cdots$, where $u[t]$ is thus a piecewise constant signal. That is, determine the $F, G, H,$ and I matrices in the following discrete-time system.

$$x[(k+1)T] = Fx[kT] + Gu[kT]$$
$$y[kT] = Hx[kT] + Iu[kT]$$

3. (20%) Consider the following system



What can you tell about the BIBO stability of the system if you only know that the input signal $u(t)$ is bounded by 10?

4. (30%) Consider the linear state equation

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t) \quad ; \quad x(t_0) = x_0 \\ y(t) = C(t)x(t) + D(t)u(t) \end{cases}$$

(a) Give the definition of *"controllable on* $[t_0, t_f]$*"*.

(b) Show that the linear state equation is *controllable on* $[t_0, t_f]$ iff the $n \times n$ *controllability Gramian*

$$W(t_0, t_f) \equiv \int_{t_0}^{t_f} \Phi(t_0, t)B(t)B'(t)\Phi'(t_0, t)dt$$

is invertible.

(c) Verify whether the following system is controllable or not.

$$\dot{x}(t) = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} x(t) + \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} u(t) \quad ; \quad t \in [t_0, t_f] \quad \text{where} \quad a_1 \neq a_2$$