

國立臺北科技大學

九十六學年第一學期電機系博士班資格考試

現代控制理論試題

填學生證號碼

第一頁 共二頁

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注意事項：

1. 本試題共【7】題，配分共140分，70分及格。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間：二小時。

1. Given a matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 5 & 3 \end{bmatrix}$, find e^{At} by (a) Caley Hamilton theorem (b) Laplace

Transform. (20%)

2. Show that if λ is an eigenvalue of A with eigenvector x , then $f(\lambda)$ is an eigenvalue of $f(A)$ with the same eigenvector x . (20%)

3. Given a transfer function matrix

$G(s) = \begin{bmatrix} \frac{5s-10}{3s+1} & \frac{-6}{s+4} \\ 3 & \frac{2s+7}{(s+4)^2} \end{bmatrix}$, find the realization by column. (20%)

4. Given a linear time invariant system (A, B, C, D) , show that the transformed system $(\bar{A}, \bar{B}, \bar{C}, \bar{D})$ obtained by the equivalence transformation from (A, B, C, D) with the nonsingular equivalence transformation matrix P has (a) the same eigenvalues as (A, B, C, D) (b) the same transformation matrix as (A, B, C, D) . (20%)

5. Show that the pair (A, C) is observable if and only if $W_o(t) = \int_0^t e^{A^T \tau} C^T C e^{A \tau} d\tau$ is nonsingular for any $t > 0$. (20%)

6. Reduce the state equation

$$\dot{x} = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \quad y = [0 \ 1 \ 1 \ 0 \ 1] x$$

to a controllable and observable state equation. Please explain your answer. (20%)

7. Given a single-input single-output system (A, B, C, D) show that the system is a minimal realization if (A, B) is controllable and (A, C) is observable. (20%)