1. (10 %) Determine reasonable membership functions for fuzzy sets "hot" and "cool."

2. (20 %) Give two examples of linguistic variable.
   Combine these linguistic variables into a compound fuzzy proposition and determine its membership function.

3. (20 %) Describe the structure of a system including a fuzzy controller. Then, modify the controller into a direct **adaptive** fuzzy controller.

4. (20 %) Consider the unforced fuzzy system model:

$$\dot{x} = \sum_{i=1}^{2} \alpha_i A_i x,$$

   where $\alpha_1,\ \alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 = 1$. Derive the stability condition (in terms of linear matrix inequalities) using Lyapunov method.

5. (30 %)

   (a) Consider the following nonlinear system:

$$\begin{aligned} \dot{x}_1 &= 3x_1 - x_2 \sin x_1 + u \\ \dot{x}_2 &= x_1 \cos x_2 + x_2 \end{aligned}$$

   Construct a T-S fuzzy system (e.g., IF $\cdots$, THEN $\cdots$.) which can represent the nonlinear system exactly.

   (b) Using the concept of PDC, design the control law u.

   (c) Briefly describe how to determine the feedback gains.