

# 國立臺北科技大學

101 學年第二學期電機系博士班資格考試

## 最佳控制 試題

第一頁 共一頁

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### 注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間：二小時。

1. (25%) Consider the function  $y = f(x) = 4 - x^{3/2}$ . Find the point on  $f(x)$  which minimizes the distance between the function and the point (3, 4).

2. (25%) Show that the Hamiltonian matrix  $M = \begin{bmatrix} F & -GR^{-1}G^T \\ -D^T D & -F^T \end{bmatrix}$  has no pure imaginary eigenvalues under the assumptions that  $(F, G)$  is controllable and  $(F, D)$  is observable.

3. For an infinite-time LQR problem, suppose the considered linear system has the Hamiltonian matrix as:

$$M = \begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

(a) (15%) Find the optimal closed-loop poles.

(b) (10%) Find the optimal control gain  $K$ , provided  $G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ .

4. (25%) Consider the scalar system  $\dot{x}(t) = x(t) + u(t)$  with the performance

$$\text{index } J = \frac{1}{2} \int_{t_0}^T (u^2 + 2x^2) dt, \quad t_0 = 0, T = 10.$$

(a) (15%) Compute the optimal linear quadratic regulator.

(b) (10%) Find the optimal cost.