國 立 臺 北 科 技 大 學

105 學年二學期電機系博士班資格考試

最佳控制 試題

注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 可使用非程式型計算機。
3. 請按順序標明題號作答，不必抄題。
4. 全部答案均須答在試卷答案欄內，否則不予計分。
5. 考試時間:二小時。

1. (25%) Consider the parallel RC circuit indicated in Figure 1 with $R=1\Omega$ and $C=1\text{H}$. Let the state variable be $x(t)=e_C(t)$ and the control current input be $u(t)=i_s(t)$. Find $u^*(t)$ to minimize the cost function $J=\frac{1}{2}x^2(2)+\frac{1}{2}\int_0^2 u^2(t)dt$.

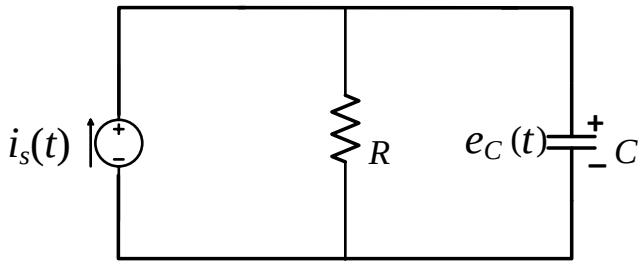

Figure 1

2. (25%) Consider the system $\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ with the performance index $J = \frac{1}{2}\int_0^\infty e^{\alpha t}\left[x_2{}^2 + u^2\right]dt$. Use the LQ control technique to find the optimal regulator u^* such that the real parts of closed-loop poles are smaller than -2.

3. (25%) Consider the dynamic system $\dot{x} = Fx + Gu + Dv$ and $y = H^T x + \omega$ where v and ω are Gaussian white noises with zero mean satisfying $E\{v(t)v^T(\tau)\} = Q\delta(t-\tau)$ and $E\{\omega(t)\omega^T(\tau)\} = R\delta(t-\tau)$.
$F = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $H^T = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $Q = 1$, and $R = 1$. Find the optimal total energy of estimation error.

4. (25%) In the following Figure 2, the cost of traveling from one node to another is given by the numbers and motion is allowed only from left to right. Find all possible minimum cost paths from node A to the desired final state K by using the principle of optimality (Do not just give answer without reasoning and the optimal cost path may not be unique).

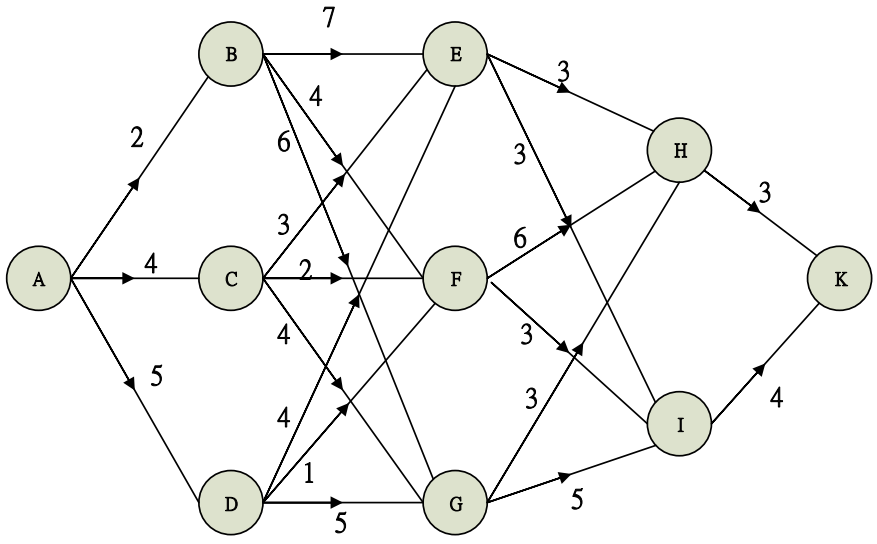

Figure 2.