國 立 臺 北 科 技 大 學

一百學年第一學期電機系博士班資格考試

最 佳 控 制 試題

注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間：二小時。

1. (25%) Show that the maximum value of $x^2 + y^2 + z^2$ on the surface $x^2 y^2 z^2 = (r^2/3)^3$ is r^2.

2. (25%) Let the plant $\dot{x}(t) = Ax(t) + Bu(t)$ where $A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The performance index is given by $J = \frac{1}{2} \int_0^\infty \left[x^T(t) Q x(t) + u^T(t) R u(t) \right] dt$ where $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = [1]$. Determine the optimal state feedback control $u(t) = -K(t)x(t)$.

3. (25%) Consider the following system

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t), \quad E\{v(t)v^T(\tau)\} = \delta(t-\tau)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t) + \omega(t), \qquad\qquad E\{\omega(t)\omega^T(\tau)\} = \delta(t-\tau)$$

where $v(t)$ and $\omega(t)$ are independent white noises with zero mean. Suppose that the estimator gain matrix $K_e = \begin{bmatrix} 8 & -14 \end{bmatrix}^T$. Calculate the steady-state covariance matrix of the estimation error.

4. (25%) Consider a discrete-time control system defined by

$$x(k+1) = x(k) + u(k), \quad x(0) = 1.$$

Determine the optimal control $u(k) = -Kx(k)$ for the performance index:

$$J = \frac{1}{2} \sum_{k=0}^{\infty} [x^2(k) + u^2(k)].$$