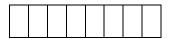
國立臺北科技大學

一百學年第二學期電機系博士班資格考試

隨機程序試題

第一頁 共二頁



- <u>注意事項</u>: 1. 本試題共【5】題,配分共100分。 2. 請按順序標明題號作答,不必抄題。 3. 全部答案均須答在試卷答案欄內,否則不予計分。
- 考試時間:二小時。
- (5pt) Give the definition of a wise-sense stationary process. 1. (a)
 - (b) (5pt) Give the definition of a random variable X on a probability space (Ω, F, P) .
 - (c) (5pt) Give the definition of a random process is a counting process.
 - (d) (5pt) Give the definition of the σ -field generated by a random variable X.
- 2. (a) (9pt) Give the definitions for a random sequence $\{X_n; n \in \mathbb{N}\}$ converging in the following modes: point wise convergence, almost sure convergence, and convergence in *r*_th mean.
 - (b) $\{X_n; n \in \mathbb{N}\}\$ is a sequence of *r.v*'s on ((0,1], B(0,1], P) where *P* is the Lebesque measure on unit interval. Let $X_n(\omega) = \begin{cases} n^{1/5} & \frac{n-1}{n} < \omega \le 1\\ 0 & \text{else} \end{cases}$.
 - (i) (3pt) Show that $\{X_n; n \in \mathbb{N}\}$ converges almost surely, but not point-wisely.
 - (ii) (3pt) Show that $\{X_n; n \in \mathbb{N}\}$ converges in r_th mean for $r \le 4$.
 - (c) (5pt) Show that if $\{X_n; n \in \mathbb{N}\}$ converges in k_th mean for some $k \in \mathbb{N}$, then $\{X_n; n \in \mathbb{N}\}$ also converges in *s*_th mean where s < k and $s \in \mathbb{N}$.
- 3. (a) (10pt) Gram-Schmidt orthogonalization procedure: Let \overline{X} be a n-dimensional random vector with mean vector $\overline{m} = [m_1 \cdots m_n]^T$ and covariance matrix $C_{\overline{x}}$. Define $\overline{Z} = [Z_1, \cdots, Z_n]^T$ by $Z_1 = X_1 - m_1$ the random vector

$$Z_{k+1} = X_{k+1} - m_{k+1} - \sum_{j=1}^{k} \frac{\operatorname{cov}(X_{k+1}, Z_j)}{\operatorname{var}(Z_j)} Z_j \text{ for } k = 1, \dots, n-1, \text{ where the fraction } 0/0 \text{ is}$$

defined as 0. Show that \overline{Z} is a 0-mean uncorrelated random vector, and there exists a lower triangular A such that $\overline{X} = A\overline{Z} + \overline{m}$.

(b) (10pt) Let
$$\overline{m} = \begin{bmatrix} 1 \ 2 \ 3 \end{bmatrix}^T$$
 and $C_{\overline{X}} = \begin{bmatrix} 0 & 3 & 1 \\ 3 & 4 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Find A in part (a).

- 4. Let $\{X_n; n=1,2,...\}$ be a random process such that $X_n = \cos(nY)$ where Y is a random variable with uniform density on $(-\pi,\pi)$.
 - (a) (4pt) Find EY and characteristic function $\phi_Y(u)$ of Y
 - (b) (9pt) Find EX_n , covariance function $k_x(m,n)$ and $E[X_n^2 X_m]$ for $\{X_n; n = 1, 2, ...\}$.
 - (c) (4pt) Is $\{X_n; n = 1, 2, ...\}$ wise-sense stationary? Explain your answer.
 - (d) (3pt) Is $\{X_n; n = 1, 2, ...\}$ stationary? Explain your answer.
- 5. Let $\{X_n; n \in \mathbb{Z}\}$ be an i.i.d. process with pdf $f_{X_n}(x) = \begin{cases} e^{-x} & x \ge 0\\ 0 & x < 0 \end{cases}$ and let $\{N_t, t \ge 0\}$ be the Poisson counting process with rate $\lambda = 2$. Define $\{X, t \ge 0\}$ by $X = \sum_{i=1}^{N_t} X_i$
 - the Poisson counting process with rate $\lambda = 2$. Define $\{Y_t, t \ge 0\}$ by $Y_t = \sum_{k=1}^{N_t} X_k$.
 - (a) (4pt) Show that the characteristic function of X_n is $\Phi_{X_n}(u) = \frac{1}{1 iu}$.
 - (b) (4pt) Find EY_t .
 - (c) (6pt) Find $var(Y_t)$
 - (d) (6pt) Find the characteristic function $\phi_{Y_t}(u)$ of Y_t .