國 立 臺 北 科 技 大 學

九十八學年第二學期電機系博士班資格考試

最佳控制 試題

注意事項：

1. 本試題共【5】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間:二小時。

1. (20%) Find the rectangle of maximum area 4*xy* that can be inscribed inside an ellipse $x^2/a^2 + y^2/b^2 = 1$.

2. (20%) Let $M = \begin{bmatrix} F^{-1} & F^{-1}GR^{-1}G^T \\ QF^{-1} & F^T + QF^{-1}GR^{-1}G^T \end{bmatrix}$ be the Hamiltonian matrix associated to the discrete-time optimal control problem, where *Q* and *R* is symmetric and positive definite. Show that if λ_i is an eigenvalues of M, then so is $\frac{1}{\lambda_i}$. (Hint: Use $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$.)

3. (20%) Let $\dot{x}_1 = u, \quad \dot{x}_2 = x_1, \quad J = \frac{1}{2}\int_0^\infty \left(x_1^2 + x_2^2 + u^2 + 2x_1u\right)dt.$
Find the optimal control law u^*.

4. (20%) Solve K_e for the following optimization problem:

$$\min_{K_e} \quad \frac{1}{2}\text{trace}(P_e)$$

subject to

$$g = F_e P_e + P_e F_e^T + K_e R K_e^T + DQD^T = 0$$

5. (20%) Consider a discrete-time control system defined by

$$x(k+1) = 0.3679x(k) + 0.6321u(k), \quad x(0) = 1$$

with the following performance index:

$$J = \frac{1}{2}[x(10)]^2 + \frac{1}{2}\sum_{0}^{9}[x^2(k) + u^2(k)].$$

Determine the optimal control law $u^*(8)$.