國立臺北科技大學

九十七學年第一學期電機系博士班資格考試

隨機程序

填學生證號碼 　　　　　　　　　　　　　　第一頁　共二頁

注意事項：

1. 本試題共【5】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間:二小時。

1. (a) (3pt) Let X be a random variable on a probability space $(\Omega, \boldsymbol{F}, P)$. Give the definition of the distribution of the random variable X.

   (b) (9pt) State and prove the basic three properties of the cdf $F_X(x) \triangleq P(X \le x)$.

   (c) (5pt) Let $\Omega = (0,1]^2, \boldsymbol{F} = B\left((0,1]^2\right)$ and P be a probability measure with uniform density $(\omega_1, \omega_2) \in (0,1]^2 \quad f(\omega_1, \omega_2) = 1$. Define the a real-valued function X on Ω by $X(\omega_1, \omega_2) = \min\{\omega_1, \omega_2\}$. Find cdf of X.

   (d) (5pt) $X_1, \cdots, X_n$ are i.i.d. random variables with common marginal cdf F_X. Let $Z = \min_{1 \le i \le n} X_i$. Find the cdf of Z.

2. Consider the probability space $((0,1], B((0,1]), P)$, where P be a probability measure with uniform density on $(0,1]$.

   (a) (6pt) Define the random variables $X(\omega) = I_{(0,1/3]}(\omega), Y(\omega) = 2I_{(0,1/3]}(\omega) + 4I_{(1/3,1]}(\omega)$ and $Z(\omega) = 3I_{(0,1/2]}(\omega) + 5I_{(1/2,1]}(\omega)$ for $\omega \in \Omega$ where $I_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases}$ is a indicator function. Let $V = X + Z$ and $W = \max\{Y, Z\}$. Find $\sigma(X)$, $\sigma(Y)$ and $\sigma(W)$.

   (b) (4pt) Find the cdfs and pmfs of V and W.

   (c) (4pt) Define a random vector $\{X_k, k \in \{1,2,\ldots,n\}\}$ on $((0,1], B((0,1]), P)$ such that $\Pr(X_k = 1) = \Pr(X_k = 0) = 1/2$ for all k and $\Pr(X_i = 1, X_j = 1) = 1/4$ for $i \ne j$.

   (d) (4pt) Show that $\{X_k, k \in \{1,2,\ldots,n\}\}$ in part (c) is a i.i.d. process.

3. A random process $\{X_t; t \geq 0\}$ such that for $0 = t_0 < t_1 < \cdots < t_n$, the joint characteristic function of $X_{t_0}, \cdots, X_{t_n}$ is given by $\phi_{X_{t_0}, \cdots, X_{t_n}}(u_0, \cdots, u_n) = \exp\left\{-\frac{\sigma^2}{2} \sum_{i=1}^{n} (t_i - t_{i-1})(u_i + \cdots + u_n)^2\right\}$.

   (a) (4pt) Find characteristic function $\phi_{X_{t_0}}(u_0)$ and cdf $F_{X_0}(x)$ of random variable X_0.

   (b) (6pt) Show that the increments $Y_1 = X_{t_1} - X_{t_0}$, $Y_2 = X_{t_2} - X_{t_1}$, ..., and $Y_n = X_{t_n} - X_{t_{n-1}}$ are independent.

   (c) (5pt) Show that the process is Gaussian and compute the covariance of $\text{cov}(X_{t_i}, X_{t_j})$.

   (d) (5pt) Is $\{X_t; t \geq 0\}$ an independent stationary increment (i.s.i) process? Justify tour answer.

4. (a) (10 pt) Give the definitions of a random sequence $\{X_n; n \in \mathbb{N}\}$ converge in the following five modes: point wise convergence, almost sure convergence, convergence in probability, convergence in r_th mean for $r \geq 1$, convergence in distribution.

   (b) (5pt) Let $((0,1], B((0,1]), P)$ be a probability space with pdf $f(\omega) = 1$; $\omega \in (0,1]$. Determine whether the sequence $\{X_n; n = 1, 2, \ldots\}$ of random variables defined by $X_n(\omega) = \sqrt{n} I_{(0,1/n]}(\omega)$ converges in any of following modes: almost sure convergence, convergence in probability, convergence in r_th mean for $r \geq 1$.

   (c) (5pt) Let $X_1, X_2, \ldots$ be i.i.d. random variables. Define $Y_n = \max_{i \leq n} X_i$. Determine whether the sequence $\{Y_n; n = 1, 2, \ldots\}$ of random variables converges in distribution.

5. (a) (5pt) Give the definition of a wise-sense stationary process.

   (b) (5pt) Give the definition of a stationary process.

   (c) (5pt) Give an example of a random process such that it is stationary and dependent. Explain your answer.

   (d) (5pt) Show that a stationary process is also a wise-sense stationary process