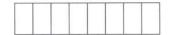
國立臺北科技大學

九十七學年第一學期電機系博士班資格考試

隨機程序

填學生證號碼

第一頁 共二頁



- 本試題共【5】題,配分共100分。
 請按順序標明題號作答,不必抄題。
 全部答案均須答在試卷答案欄內,否則不予計分。
- 1. (a) (3pt) Let X be a random variable on a probability space (Ω, F, P) . Give the definition of the distribution of the random variable X.
 - (b) (9pt) State and prove the basic three properties of the cdf $F_X(x) \triangleq P(X \le x)$.
 - (c) (5pt) Let $\Omega = (0,1]^2$, $F = B((0,1)^2)$ and P be a probability measure with uniform density $(\omega_1, \omega_2) \in (0,1]^2$ $f(\omega_1, \omega_2) = 1$. Define the a real-valued function X on Ω by $X(\omega_1, \omega_2) = \min\{\omega_1, \omega_2\}$. Find cdf of X.
 - (d) (5pt) X_1, \dots, X_n are i.i.d. random variables with common marginal cdf F_X . Let $Z = \min_{1 \le i \le n} X_i$. Find the cdf of Z.
- 2. Consider the probability space ((0,1], B((0,1]), P), where P be a probability measure with uniform density on (0,1].
 - (a) (6pt) Define the random variables $X(\omega) = I_{(0,1/3]}(\omega), Y(\omega) = 2I_{(0,1/3]}(\omega) + 4I_{(1/3,1]}(\omega)$ and $Z(\omega) = 3I_{(0,1/2]}(\omega) + 5I_{(1/2,1]}(\omega) \quad \text{for} \quad \omega \in \Omega \quad \text{where} \quad I_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases} \quad \text{is a indicator}$ function. Let V = X + Z and $W = \max\{Y, Z\}$. Find $\sigma(X)$, $\sigma(Y)$ and $\sigma(W)$.
 - (b) (4pt) Find the cdfs and pmfs of V and W.
 - (c) (4pt) Define a random vector $\{X_k, k \in \{1, 2, ..., n\}\}\$ on $\{(0, 1], B((0, 1]), P\}$ such that $Pr(X_k = 1) = Pr(X_k = 0) = 1/2$ for all k and $Pr(X_i = 1, X_j = 1) = 1/4$ for $i \neq j$.
 - (d) (4pt) Show that $\{X_k, k \in \{1, 2, ..., n\}\}$ in part (c) is a i.i.d. process.

- 3. A random process $\{X_t; t \geq 0\}$ such that for $0 = t_0 < t_1 < \dots < t_n$, the joint characteristic function of X_{t_0}, \dots, X_{t_n} is given by $\phi_{X_{t_0}, \dots, X_{t_n}}(u_0, \dots, u_n) = \exp\left\{-\frac{\sigma^2}{2} \sum_{i=1}^n (t_i t_{i-1})(u_i + \dots + u_n)^2\right\}.$
 - (a) (4pt) Find characteristic function $\phi_{X_{t_0}}(u_0)$ and cdf $F_{X_0}(x)$ of random variable X_0 .
 - (b) (6pt) Show that the increments $Y_1=X_{t_1}-X_{t_0}$, $Y_2=X_{t_2}-X_{t_1}$, ..., and $Y_n=X_{t_n}-X_{t_{n-1}}$ are independent.
 - (c) (5pt) Show that the process is Gaussian and compute the covariance of $cov(X_{t_i}, X_{t_i})$.
 - (d) (5pt) Is $\{X_t; t \ge 0\}$ an independent stationary increment (i.s.i) process? Justify tour answer.
- 4. (a) (10 pt) Give the definitions of a random sequence $\{X_n; n \in \mathbb{N}\}$ converge in the following five modes: point wise convergence, almost sure convergence, convergence in probability, convergence in r_th mean for $r \ge 1$, convergence in distribution.
 - (b) (5pt) Let ((0,1], B((0,1]), P) be a probability space with pdf $f(\omega) = 1$; $\omega \in (0,1]$. Determine whether the sequence $\{X_n; n = 1, 2, \ldots\}$ of random variables defined by $X_n(\omega) = \sqrt{n} I_{(0,1/n]}(\omega)$ converges in any of following modes: almost sure convergence, convergence in probability, convergence in r th mean for $r \ge 1$.
 - (c) (5pt) Let $X_1, X_2,...$ be i.i.d. random variables. Define $Y_n = \max_{i \le n} X_i$. Determine whether the sequence $\{Y_n; n = 1, 2, ...\}$ of random variables converges in distribution.
- 5. (a) (5pt) Give the definition of a wise-sense stationary process.
 - (b) (5pt) Give the definition of a stationary process.
 - (c) (5pt) Give an example of a random process such that it is stationary and dependent. Explain your answer.
 - (d) (5pt) Show that a stationary process is also a wise-sense stationary process