國 立 臺 北 科 技 大 學

九十八學年第一學期電機系博士班資格考試

現代控制理論試題

注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。
4. 考試時間:二小時。

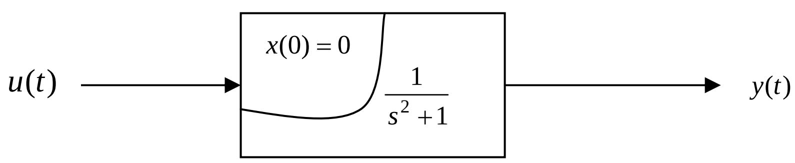
1. (20%) Consider the following linear time-invariant system.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Give the discretized system for the case $u[t] = u[kT]; t \in [kT, (k+1)T), k = 0,1,2,\cdots$, where $u[t]$ is thus a piecewise constant signal. That is, determine the $F, G, H,$ and I matrices in the following discrete-time system.

$$x[(k+1)T] = Fx[kT] + Gu[kT]$$
$$y[kT] = Hx[kT] + Iu[kT]$$

2. (20%) Consider the following system



What can you tell about the BIBO stability of the system if you only know that the input signal $u(t)$ is bounded by 10?

3. (30%) Consider the linear time-invariant state equation

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

Suppose that the controllability matrix for (1) satisfies

$$rank[B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B] = q \quad ; \quad 0 < q < n.$$

Show that there exists an invertible $n \times n$ matrix P such that

$$P^{-1}AP = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ 0_{(n-q)\times q} & \hat{A}_{22} \end{bmatrix} \quad ; \quad P^{-1}B = \begin{bmatrix} \hat{B}_{11q\times m} \\ 0_{(n-q)\times m} \end{bmatrix}$$

and

$$rank[\hat{B}_{11} \quad \hat{A}_{11}\hat{B}_{11} \quad \hat{A}_{11}^2\hat{B}_{11} \quad \cdots \quad \hat{A}_{11}^{q-1}\hat{B}_{11}] = q.$$

4. (30%) Consider the unforced state equation

$$\begin{cases} \dot{x}(t) = A(t)x(t) \quad ; \quad x(t_0) = x_0 \\ y(t) = C(t)x(t) \end{cases}$$

(a) Give the definition of "*observable on* $[t_0, t_f]$".

(b) Show that the linear state equation (1) is *observable on* $[t_0, t_f]$ iff the $n \times n$ *observability Gramian*

$$M(t_0, t_f) = \int_{t_0}^{t_f} \Phi'(t, t_0)C'(t)C(t)\Phi(t, t_0)dt$$

is invertible.